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Research and modeling of wear of driving and rolling wheel sets in the ELF I electric multiple unit

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The aim of the article is to present the results of research and modelling of the wheelset to wear on the rolling diameter of the drive and rolling wheels of a multi-unit traction unit. The object of the research was the PESA ELF I electric multiple units constructed as a five-unit unit based on two end drive bogies and three central rolling bogies. The article presents the results of research on the change in wheel diameter recorded every two months during a periodic inspection over a period of five years of vehicle operation until revision repair. Based on the results of wheel diameter tests, wear was determined, on the basis of which the change in wheel diameter and wear as a function of time and vehicle mileage was modelled. Based on the models, mileage expressed in kilometres was determined for each wheelset with longer vehicle operation. The reason for implementing the subject is the uneven wear of individual wheels in the wheelset of drive and rolling bogies in multi-unit traction units. The vehicle maintenance system documentation requires that after a specified period of time, e.g. 5 years, the vehicle is sent for a revision repair, even if none of the wheel sets has reached the wear limit.

Key words: *wheel diameter, wheel wear, electric multiple unit, operational tests*

1. Introduction

Analyzing the scope of work performed during the operation of electric bogies or diesel traction units, it is found that the most common damage occurs in the running gear, i.e. in wheelsets [29, 30]. For this reason, the most time is spent on performing the necessary geometric measurements of the wheelset, filling in the appropriate measurement cards in accordance with the template included in the DSU maintenance system documentation for a given vehicle, and in the case of wear and exceeding any parameter of the wheelset – sending it for repair by turning or replacing it with a new one [31]. Other components located on the bogie, such as traction motors, mechanical transmissions, spring systems or wheelset guidance systems, are characterized by lower wear intensity according to the works [23, 24] than the wheelset rolling on the rail. According to the above-mentioned reliability function R(t) as a function of operating time, in the works of Piec, it was found that in each period, it is possible to rank the reliability of the components (assemblies) of the rolling bogie in the following order: frame, pivot pins, suspension and guidance system, brake system and the most unreliable wheelset. The identified cases of sudden damage in the drive system or cracks, e.g. in the area of bogie frames or other bogie components, were random and could result from system errors made during the design and construction of the bogies or from negligence during inspections and repairs of the components located on the rolling or driving bogies [9]. In traction units, depending on the design speed, the number of members with driving and driving bogies as well as the load of individual wheelsets, the process of wheel tread wear does not proceed evenly on all wheelsets [8]. The above-mentioned factors influencing the wear of wheelsets result from the design features of the vehicle. The second group of factors that affect the intensity of wear of the wheel rim surface not related to the vehicle is the terrain (mountainous or lowland areas), the number of bends, as well as climatic conditions, i.e. driving in winter, autumn or summer. Another reason for increased wheel wear is the creation of a flat spot on the wheel as a result of braking on blocked wheels. In this place, due to the short-term effect of high temperature, a transformation in the wheel structure into hard martensite occurs [19, 33]. As a consequence, it is necessary to turn the wheelset between periodic inspection cycles and reduce the wheel diameter.

Taking into account the safety criterion against derailment, it is stated that the friction coefficient μ and the flange angle γ affect the wheel flange wear. Based on many years of research described in [3, 14, 41], the flange angle of 70° was assumed, and the friction coefficient of 0.36.

The analysis of friction phenomena and wear processes in the surface layer of railway wheels as a result of contact with the rail or additionally with the brake pad shows that the complexity of friction processes that occur as a result of wheels rolling on rails causes difficulties in building a general model of the intensity of wear of the rolling surface that takes into account all factors accompanying the friction process. There is a need for further research to make it possible to obtain such models. From a practical point of view, it is also justified to take such actions aimed at maximally extending the service life of the wheelset, also described in [1, 18].

In the field of modelling the wear of railway wheels, the authors in work [10] used the machine learning technique (ELM) based on the optimization algorithm (PSO) to investigate the effect of the difference in wheel diameters on the change of the hardness of the rolling surface. In the field of multi-angle wheel wear, the authors in the work [37] proposed an elastic model of the coupled wheel-rail system, in which the railway wheel is decoupled from the vibrations of the wheel rolling on the rail. Other authors in the work [42] presented a wheel wear model taking into account the flexibility of the wheelset and the track as a result of the acoustic roughness of the wheels, on the example of subway vehicles in China.

2. Research objective and methodology

The research problem presented in the article concerns uneven wear of wheels of wheel sets. The thesis was put forward that wheel wear is influenced by both the purpose of the wheelset (driving or rolling) and its location in the vehicle. The research method used to solve this problem is based on a passive experiment. Based on the documentary data from the maintenance of the vehicle used in a given period, the characteristics were determined as a function of the vehicle's mileage and time of use. The measuring tools were measuring instruments used during vehicle maintenance, such as a wheel diameter gauge and a wheel profile caliper [5]. The aim of the article is to present the results of research and modelling of the change in wheel diameter and wear, wheelsets on the driving and rolling bogies of a multi-unit traction unit. During the research, only one parameter was recorded, i.e. wheel diameter D. The remaining wheel profile parameters were not analysed or assessed because the wheel diameter was the most frequently exceeded parameter found during the P2 inspection in relation to the remaining geometric parameters of the wheelset, which was also confirmed in [27]. Exceeding the diameter dimension D should be explained by a difference in dimension D between the right and left wheel greater than 1 mm [25].

According to the documentation of the vehicle DSU maintenance system, every two months the vehicle was sent for a periodic P2 inspection during which, among other things, the wheel diameter D was recorded. The research was conducted for a period of 5 years when the vehicle was sent for a revision repair, during which the driving and rolling wheelsets had their wheels replaced with new ones.

The research was of a small nature of a passive experiment [7, 34], no boundary conditions were determined before the research because the electric multiple unit ran on different routes in scheduled traffic, as shown in Table 1.

Table 1. Routes covered by the tested EN76 series vehicles

Start and end station	Line length in km
Poznań Główny–Zbąszynek	81
Poznań Główny–Gniezno–Mogilno	80
Poznań Główny–Konin–Kłodowa–Kutno	179

Due to the planned traffic, wheel wear was not analyzed in terms of the route in terms of the number of curves and their radii. The analyzed vehicle ran in lowland areas, with a negligible number of small radii (less than 300 m, except for switches), which did not intensify wear in the area of the wheel flange.

2.1. Research object

The research object was the electric multiple unit EN76 ELF (Fig. 1) manufactured by the PESA Rail Vehicles Factory in Bydgoszcz. The name ELF is an abbreviation of the English words electric low floor. The vehicle premiered at the InnoTrans International Railway Fair in Berlin in 2010 [2].

Fig. 1. View of the EN76 vehicle leaving Poznań Główny station [photo by A.M. Rilo Cañás]

PESA Elf electric multiple units have been designed for various purposes. Depending on the range of their operation, the vehicles are used in agglomeration, regional and interregional traffic [11, 36]. The vehicles, in their basic versions, are powered by a 3 kV DC voltage and are designed for a track with a gauge of 1435 mm. All vehicles, regardless of type, can move at a speed of 160 km/h. The axle arrangement of the tested vehicles is Bo’2’2’2’Bo. The vehicle sections are connected to each other by a system of Jacobs rolling bogies, while the outer sections are additionally supported by driving bogies [20, 21].

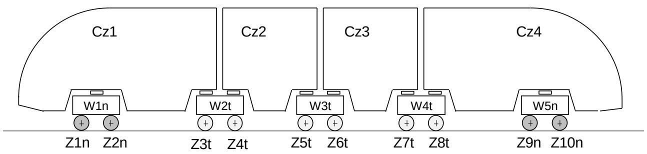

Fig. 2. EN76 vehicle diagram: Cz 1, Cz 4 – Drive unit with control cabin, Cz 2, Cz 3 – Central rolling unit, W1n, W5n – Drive bogies, W2t, W3t, W4t – Rolling bogies

The axle arrangement of the EN76 electric multiple unit is shown in Fig. 2. Figure 3 shows a view of the wheelsets with brake discs mounted on the wheels of the driving and rolling bogies.

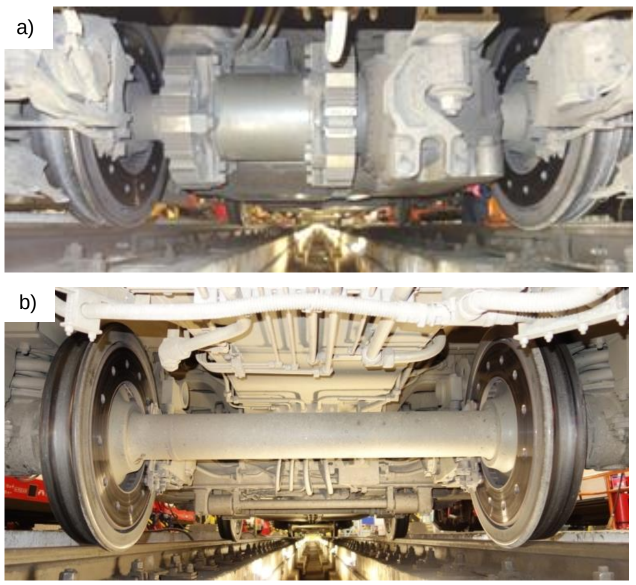

Fig. 3. View of the wheelsets of the EN76 vehicle: a) driven on a 26MNf bogie, b) rolling on a 40ANf bogie [photo by AM. Rilo Cañás]

The basic technical parameters of the PESA Elf EN76 vehicle are presented in Table 2.

Table 2. Basic technical data of the EN76 vehicle [12]

Name	Value	Unit
Total length of the unit	75,250	mm
Width of the unit	2883	mm
Height of the unit	4280	mm
Power of the traction unit	4×500	kW
Maximum operating speed	160	km/h
Service weight	134.5	t
Diameter of new and used wheels	850/780	mm/mm

The vehicle is equipped with a KE-type brake from Knorr-Bremse; the main braking is performed with an ED electrodynamic brake with an EP electro-pneumatic brake on the driving bogies and an electro-pneumatic brake on the rolling bogies [4].

2.2. Wheel diameter test results

During the tests, special attention was paid to the wheel diameter D dimension on the drive and rolling sets. First, the maximum and minimum wheel diameter dimensions were determined for all wheelsets for further analysis. The wheel diameter was monitored until the revision repair. The first objective of the tests was to determine the characteristics of the maximum and minimum diameter dimensions. Figure 4 graphically presents the relationship between the maximum and minimum dimensions recorded on all 10 wheelsets in the range of wheel diameter as a function of operating time.

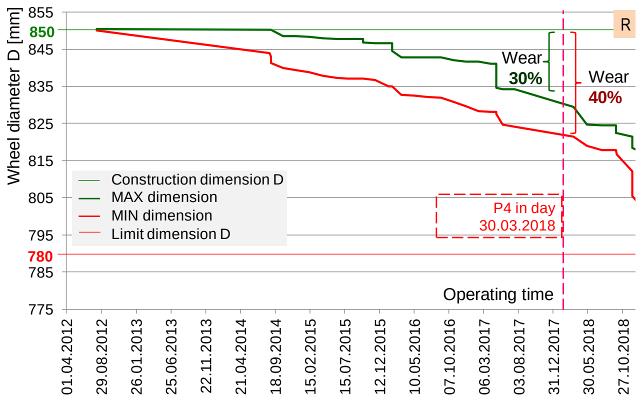

Fig. 4. Dependence of the wheel diameter in the rolling circle on the operating time, observed during P2 inspections to P4 inspections, R – right wheels

Analyzing the data presented in Fig. 4, it is found that since the first periodic inspections, differences have been observed in both diameters. The differences between the maximum and minimum dimensions reach, in extreme cases, the values of 13.1 mm for left wheels and 13.3 mm for right wheels, with average values for the 5-year period of 9.55 and 9.33 mm. According to the EN76 vehicle maintenance system documentation [11], the difference in the diameter of wheelsets cannot exceed 15 mm in the case of bogies located in subsequent vehicle sections, 5 mm in the case of one bogie and cannot exceed 1 mm in relation to the right and left wheels of the same wheelset.

Table 3. Numbers and frequencies for the maximum and minimum dimensions of the wheel diameter of wheelsets

Left wheels number	Strength $li_{\_max}$ for D	Relative frequency $pi_{\_max}$ for D	Strength $li_{\_min}$ for D	Relative frequency $pi_{\_min}$ for D
1	0	0	6	0.200
2	0	0	4	0.133
3	2	0.061	0	0
4	5	0.152	0	0
5	16	0.485	0	0
6	1	0.030	0	0
7	2	0.061	0	0
8	7	0.212	0	0
9	0	0	5	0.167
10	0	0	15	0.500
∑	**33**	**1**	**30**	**1**
Right wheels number	Strength $li_{\_max}$ for D	Relative frequency $pi_{\_max}$ for D	Strength $li_{\_min}$ for D	Relative frequency $pi_{\_min}$ for D
1	0	0	2	0.083
2	0	0	3	0.125
3	0	0	0	0
4	6	0.162	0	0
5	17	0.459	0	0
6	5	0.135	0	0
7	6	0.162	0	0
8	3	0.081	0	0
9	0	0	3	0.125
10	0	0	16	0.667
∑	**37**	**1**	**24**	**1**

The second objective of the study was to the distribution of wheelsets with the maximum and minimum dimensions of the D parameter. The aforementioned distribution was prepared as a histogram of the number of cumulative wheel diameters D for all wheelsets of the analysed EN76 vehicle. On this basis, a ranking of wheelsets was determined in which the maximum dimension of the wheel diameter, indicating the lowest wear and the minimum dimension (the greatest wheel wear) were most frequently found during P2 inspections. In Table 3, the strength of occurrence of a given maximum and minimum dimension and the relative frequencies were determined in accordance with [16].

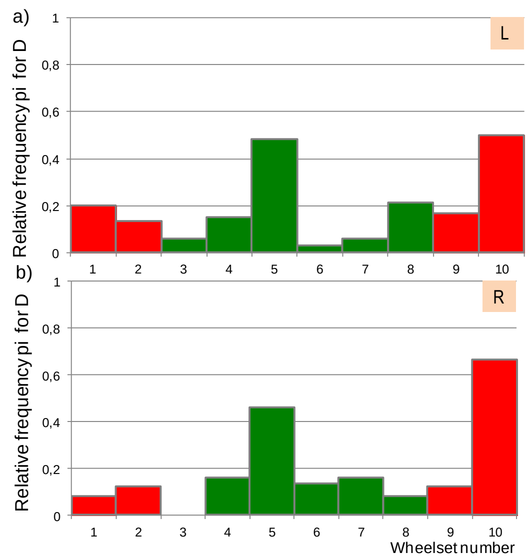

Fig. 5. Distribution of relative frequency of occurrence of the maximum (green) and minimum (red) dimensions of wheel diameter D of wheelsets during P2 inspections for the EN76 vehicle

Figure 5 graphically presents the frequency of occurrence of the maximum and minimum dimensions of the wheel diameters on all wheelsets. The minimum dimension is marked in red, and the maximum dimension of the wheelset diameter D is marked in green.

Analyzing the results in Table 3 and graphically in Fig. 5 in terms of the number and frequency of occurrence of the maximum and minimum wheel diameter during the P2 level inspection, it was found that in each case, the drive sets (first and last bogie) showed the greatest wear. This was found because, during each P2 inspection, wheelsets 1, 2, 9 and 10 had the smallest wheel diameters. The rolling sets (2, 3 and 4 bogies) showed the lowest wear because, during the inspections, they had the largest diameters in relation to the drive sets. The lack of data on the relative frequency graph means that a given wheelset never reached the maximum and minimum dimensions during the P2 periodic inspections every two months. The aforementioned wear is defined indirectly based on the number of detected cases of the lowest wheel diameter D during periodic inspections. Also, during these analyses, it was found that not in every case the extreme sets (first and last) had the greatest wear.

3. Wheel wear test results

The wheel wear on all wheelsets (drive and rolling) should be determined based on the characteristics of the wheel diameter change as a function of the operating time and mileage of the EN76 PESA ELF vehicle. Figure 6 shows the cumulative course of the wheel diameter change of the analyzed electric multiple unit.

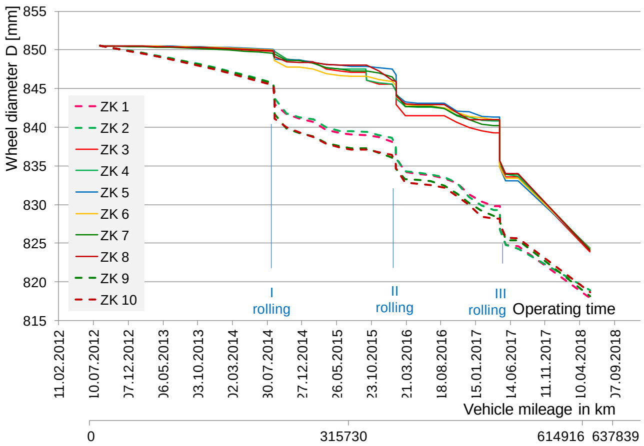

Fig. 6. Summary of the change in the diameter of the left wheels of the EN76 vehicle

Analyzing the summary graph presented in Fig. 6 regarding the change in the diameter of the (left) EN76 electric multiple units wheels, it is found that in the case of drive sets (ZK 1, ZK 2, ZK 9 and ZK 10), the decrease in the wheel diameter is greater over time compared to the rolling sets (ZK 3, ZK 4, ZK 5, ZK 6, ZK 7 and ZK 8). On this basis, it is found that the intensity of wear of drive sets is greater than that of the rolling sets. The time axis of operation additionally presents the vehicle mileage, on which only 3 values are marked. This results from the fact that for the EN76 vehicle, the maintenance system documentation to the P2 maintenance level does not provide a limit on the number of kilometres travelled until the next inspection but only refers to the time of use (driving times). The three mileage values were recorded randomly by the employee performing the measurement of geometrical quantities. An additional graph of the change in the diameter of the right wheels was not presented because, according to the DSU, the difference in the diameters of the right wheels relative to the left wheels cannot exceed 1 mm. Hence, the summary graphs would be similar. Based on the data collected from wheel diameter measurements, wheel wear was calculated for the EN76 026 vehicle. Based on the data, it was found that with the increase in the vehicle mileage and time of its use, the dimensions of wheel rolls of wheelsets and wear increase. Wheel wear is not the same, in summary, the total values of wear on individual wheelsets differ significantly from each other. Figure 7 shows the values of total wear after five years of use and the values of the summed rolls on individual wheels of the wheelset.

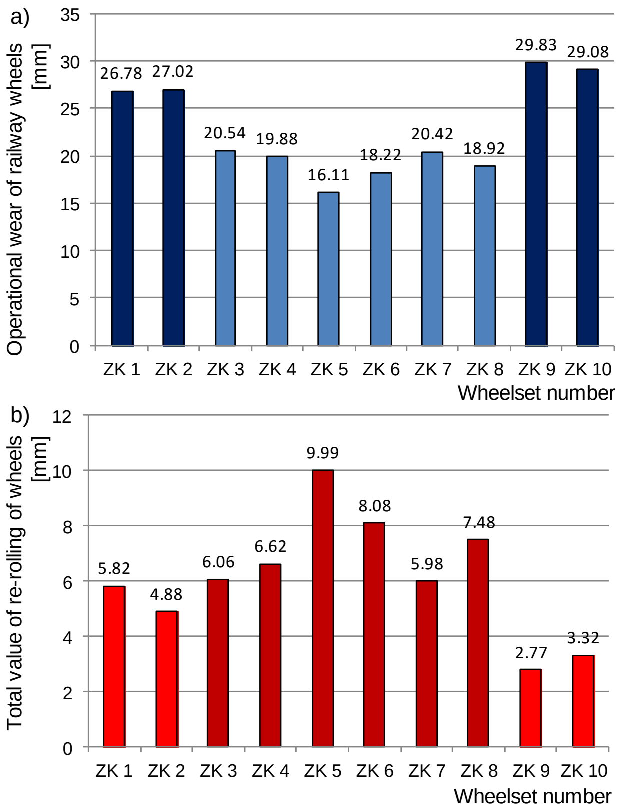

Fig. 7. Total value of: a) linear wear of wheels, b) wheel wear on individual wheel sets of the vehicle EN76

Analyzing the graphs presented in Fig. 7, it was found that in terms of wear, the drive sets showed the highest values (in the order ZK 9, ZK 10, ZK 2 and ZK 1) in relation to the rolling sets. However, in the case of the values of wheel turning on the wheelset lathe, the rolling sets were cut at a greater depth.

Table 4. Average wheel wear values on the drive and rolling sets of the EN76 026 vehicle after 5 years of operation until the P4 inspection

		Average wear value
Driven wheelsets	ZK 1, ZK 2, ZK 9 and ZK 10	32.4 mm
Rolling wheelsets	ZK 3, ZK 4, ZK 5, ZK 6, ZK 7 and ZK 8	26.4 mm
Difference between drive and rolling 6 mm		6 mm

Figure 8 shows the total wear values of railway wheels on the diameter, taking into account the values of re-rolling for the P4 inspection, i.e. for revision repair. Table 4 shows the values of average wheel wear on wheelsets of driving and rolling bogies.

Based on Fig. 8 and the average wear values of wheels and discs presented in Table 4, it was found that there is a difference between the rolling and driving sets in each case. In the case of railway wheels, the wear is lower on the rolling sets than on the driving sets.

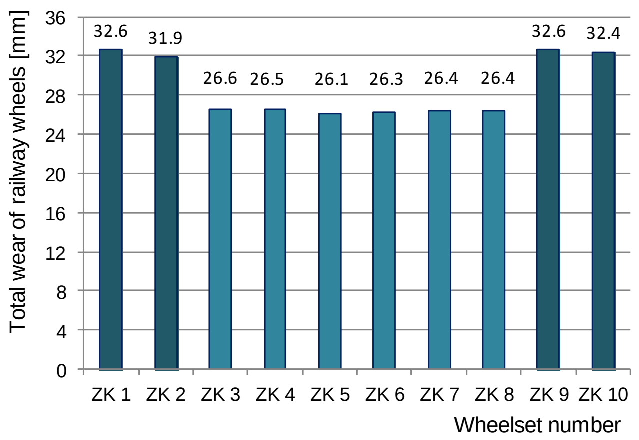

Fig. 8. Total wear and tear value of railway wheels for P4 inspection (revision repair) of the EN76 vehicle

The wear intensity describes the speed of the wheel wear process as the dependence of the mass or volume loss on the friction path. In the literature on wear processes in the construction and operation of machines, the most commonly used dependence (1) on the wear intensity is [28]. This is the volumetric wear resistance (in relation to the friction path), assuming constant reference values defined by the coefficients [26].

$$I_{OZ} = \frac{U}{L} = c\frac{N \cdot d \cdot x}{H \cdot y}\left[\frac{m^3}{m}\right] \quad (1)$$

where: U – volume loss of material [m^3], L – friction path [m], N – unit pressure (normal load), c – proportionality factor, d – diameter of abrading particles, measured perpendicularly to the direction of movement, x – ratio of the number of abrading particles to the total number of particles, H – Vickers hardness, y – coefficient of the relative number of particles carrying the load.

In practice (in road or rail vehicles) the wear intensity refers to the vehicle mileage or time of its use. Based on the analyzed vehicle in terms of wheel wear on the rolling diameter, time and vehicle mileage, the wheel wear intensity I_K was determined in Table 5.

The content of Table 5 allows for the assessment of the wear intensity of individual wheelsets in relation to one day and one kilometre of driving of the analysed vehicle. However, according to publications [13, 40], the wear intensity of the sets for practical reasons refers to 100,000 km. It should be emphasised that the wear intensity of vehicles according to [17, 38] is influenced by factors such as driving technique (vehicle use), quality of service, selection of vehicle construction materials, unit pressures and dynamic load, precision of machining with assembly and type of vehicle construction. The first mentioned factors refer to operational processes, while the second group of factors includes design, manufacturing technology and selection of materials [35]. This clearly shows that the person using the vehicle has the same influence on the rate of wear processes as the vehicle designer, constructor and its contractor.

Table 5. The values of the wear intensity of the I_K wheels on the drive and rolling sets of the analyzed EN76 vehicle after 5 years of operation until the P4 inspection

	In relation to time	Unit
ZK 1	**$1.495 \cdot 10^{-2}$**	[mm/day]
ZK 2	$1.463 \cdot 10^{-2}$	
ZK 3	$1.220 \cdot 10^{-2}$	
ZK 4	$1.216 \cdot 10^{-2}$	
ZK 5	$\underline{1.197 \cdot 10^{-2}}$	
ZK 6	$1.206 \cdot 10^{-2}$	
ZK 7	$1.211 \cdot 10^{-2}$	
ZK 8	$1.211 \cdot 10^{-2}$	
ZK 9	**$1.495 \cdot 10^{-2}$**	
ZK 10	$1.486 \cdot 10^{-2}$	
	In relation to the course	Unit
ZK 1	**$5.111 \cdot 10^{-5}$**	[mm/km]
ZK 2	$5.001 \cdot 10^{-5}$	
ZK 3	$4.170 \cdot 10^{-5}$	
ZK 4	$4.154 \cdot 10^{-5}$	
ZK 5	$\underline{4.091 \cdot 10^{-5}}$	
ZK 6	$4.123 \cdot 10^{-5}$	
ZK 7	$4.139 \cdot 10^{-5}$	
ZK 8	$4.139 \cdot 10^{-5}$	
ZK 9	**$5.111 \cdot 10^{-5}$**	
ZK 10	$5.080 \cdot 10^{-5}$	
Bold, e.g. **$5.111 \cdot 10^{-5}$** is the highest intensity of wheel wear, Underline, e.g. $\underline{1.259 \cdot 10^{-5}}$ is the lowest intensity of wheel wear.		

4. Wheel wear modelling

In the construction or operation of machines as well as in mechanical or electric vehicles, according to [6], one-dimensional linear models with one input and one output are very often used based on observations of the tribological system of a railway wheel. One-dimensional non-linear models (polynomial, exponential, power or logarithmic) are also used. The selection of models for changing the wheel diameter D or their wear is made based on the highest value of the coefficient of determination R^2 of the model fit to the values from the measurements. Then, the coefficients of a given model are assessed. If the value of significance F for individual regression coefficients is greater than 0.05 according to [22], it is considered that the model coefficients are statistically insignificant. Then the model can be written in a simplified way. This applies in particular to polynomial models. Considering that the last period of wheel use is important from the point of view of estimating the further mileage of the vehicle, a quadratic model was used. The graphs presented in Fig. 9 and 10 show the values from the tests and the functions approximating the changes in the diameter of the individual wheels of the drive and rolling bogies as a function of the course. For better visibility of the models and their course, different colors were intentionally used on the graphs.

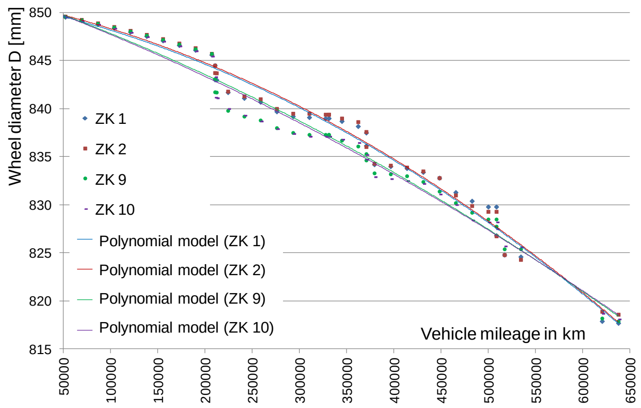

Fig. 9. Dependence of wheel diameter in the rolling circle D for drive sets ZK 1, ZK 2, ZK 9 and ZK 10 as a function of vehicle mileage with regression quadratic models

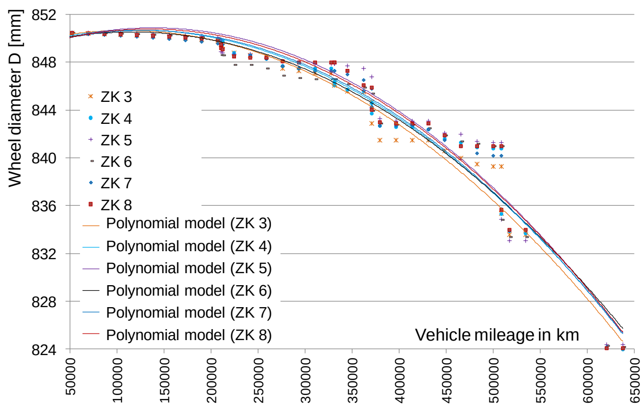

Fig. 10. Dependence of wheel diameter in the rolling circle D for rolling sets ZK 3, ZK 4, ZK 5, ZK 6, ZK 7 and ZK 8 as a function of vehicle mileage with regression quadratic models

The models of the change in the diameter of the drive gears as a function of the mileage are presented by the following equations:

$$D_{ZK1\_N} = -4.87 \cdot 10^{-11}x^2 - 2.07 \cdot 10^{-5}x + 8.50 \cdot 10^2 \quad (2)$$

$$D_{ZK2\_N} = -4.86 \cdot 10^{-11}x^2 - 2.08 \cdot 10^{-5}x + 8.51 \cdot 10^2 \quad (3)$$

$$D_{ZK9\_N} = -2.74 \cdot 10^{-11}x^2 - 3.45 \cdot 10^{-5}x + 8.51 \cdot 10^2 \quad (4)$$

$$D_{ZK10\_N} = -2.44 \cdot 10^{-11}x^2 - 3.62 \cdot 10^{-5}x + 8.51 \cdot 10^2 \quad (5)$$

models of changing the diameter of the wheels of the rolling sets are:

$$D_{ZK3\_T} = -9.28 \cdot 10^{-11}x^2 + 2.01 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (6)$$

$$D_{ZK4\_T} = -9.47 \cdot 10^{-11}x^2 + 2.27 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (7)$$

$$D_{ZK5\_T} = -10.02 \cdot 10^{-11}x^2 + 2.82 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (8)$$

$$D_{ZK6\_T} = -9.08 \cdot 10^{-11}x^2 + 2.09 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (9)$$

$$D_{ZK7\_T} = -9.78 \cdot 10^{-11}x^2 + 2.51 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (10)$$

$$D_{ZK8\_T} = -9.99 \cdot 10^{-11}x^2 + 2.68 \cdot 10^{-5}x + 8.49 \cdot 10^2 \quad (11)$$

The coefficient of determination for models described by equations (2)–(11) of wheel diameter changes as a function of mileage was 0.95–0.98. Analyzing the graphs presented in Fig. 9 and 10 with models describing wheel diameter changes as a function of vehicle mileage, it can be seen that in the case of drive sets, characteristics close to a linear function were obtained with lower values of the directional coefficients (2–$5 \cdot 10^{-11}$) at x^2, which indicates a greater inclination of the function to the axis of the vehicle mileage, in contrast to the characteristics for rolling sets. In this case, the values of the directional coefficients at x^2 are higher in the range of 9–$10 \cdot 10^{-11}$, and the courses of the mild quadratic function run in the first phase towards the axis of the analyzed vehicle mileage.

The use of non-linear (quadratic) regression models describing wheel diameter changes as a function of vehicle mileage also resulted from the need to estimate the vehicle mileage until the wheelset reaches the limit dimension of the EN76 vehicle wheel diameter. For this purpose, based on the determined quadratic models, it was necessary to determine the determinants of quadratic equations Δ ($\Delta = b^2 - 4 \cdot a \cdot c$) in accordance with the relationship and the zeros assuming that $\Delta > 0$ based on the following formulas (12) [32].

$$x_{1\_0} = \frac{-b-\sqrt{\Delta}}{2a},\ x_{2\_0} = \frac{-b+\sqrt{\Delta}}{2a} \quad (12)$$

The zeros of the quadratic function described by equations (12) refer to the value 0 on the axis of the wheel diameter D. In order to estimate the value of the course for the boundary dimension, the zeros for this dimension had to be calculated. A system of equations described by the relationship (13) was determined for the exemplary driving wheelset ZK 1. For the new quadratic function equation, the zeros were calculated at the intersection with the constant function y = 780.

$$\begin{cases} D_{ZK1} = -4.87 \cdot 10^{11}x^2 - 2.07 \cdot 10^{-5}x + 850.71 \\ D = 780 \end{cases}$$

$$\begin{cases} 4.87 \cdot 10^{11}x^2 + 2.07 \cdot 10^{-5}x - 70.71 = 0 \\ \Delta = (2.07 \cdot 10^{-5})^2 - 4 \cdot 4.87 \cdot 10^{-11} \cdot (-70.71) \\ \qquad = 1.42 \cdot 10^{-8};\ \Delta > 0 \end{cases} \quad (13)$$

$$\begin{cases} x_{1\_780} = \dfrac{-2.07 \cdot 10^{-5} - \sqrt{1.42 \cdot 10^{-8}}}{2 \cdot 4.87 \cdot 10^{-11}} = 1010534.04 \text{ km} \\ x_{2\_780} = \dfrac{-2.07 \cdot 10^{-5} + \sqrt{1.42 \cdot 10^{-8}}}{2 \cdot 4.87 \cdot 10^{-11}} = -1437110.34 \text{ km} \end{cases}$$

The limit dimension of the wheel diameter of the electric multiple unit, which is 780 mm, was inserted into all quadratic equations (2)–(11). Next, using the relations (12) and (13), the value of the zero position in the first quadrant of the coordinate system was calculated, which simultaneously corresponds to the vehicle mileage in km for the analyzed wheelset.

Table 6 contains the mileage for all 10 wheelsets to reach the limit dimension of the wheel diameter.

Analyzing the estimated values of the mileage that the electric multiple unit would achieve based on the use of quadratic regression models in the wheel diameter D range presented in Table 6, it is concluded that the wheelsets would allow the vehicle to be used for about 1 million kilometres until the limit dimension is reached on the wheel diameter of 780 mm. Due to the entries in the vehicle's DSU, which, in addition to the vehicle's mileage, also imposes a time limit, the vehicle was sent for a P4 inspection with a mileage of over 600 thousand km.

Table 6. Estimated values of the mileages that the electric multiple unit would achieve when using wheelsets to the boundary dimension based on quadratic regression models

Wheelset designation	Estimated mileage of the wheelset in [km]	Diameter boundary dimension in [mm]	Construction dimension of the new wheel diameter in [mm]
ZK 1	1010534.04	780	850
ZK 2	1011626.40		
ZK 3	980943.23		
ZK 4	984734.16		
ZK 5	**971012.40**		
ZK 6	996818.29		
ZK 7	978439.23		
ZK 8	975848.10		
ZK 9	1105246.53		
ZK 10	1124600.34		

Based on the results of the wheel diameter D measurements, modeling of wheel wear ZK was also carried out. Similarly to the modeling of the diameter as a function of mileage, wear was also modeled as a function of the vehicle's mileage using non-linear regression models. The choice of models resulted from obtaining the highest value of the determination coefficient R^2 [39].

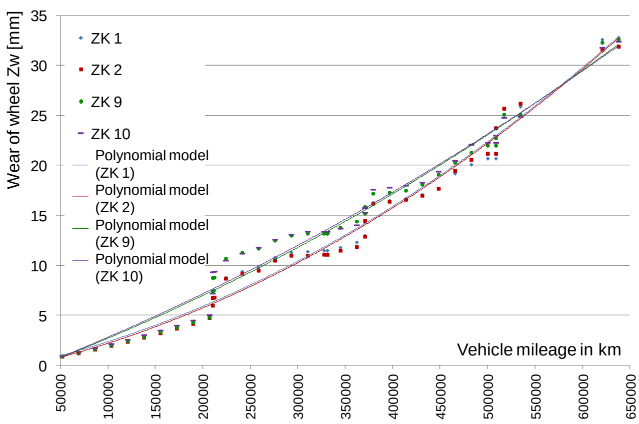

Fig. 11. Dependence of wear on the wheel diameter in the rolling circle D for the ZK 1, ZK 2, ZK 9 and ZK 10 drive sets as a function of the vehicle mileage with quadratic regression models

Figures 11–12 show the dependence of wear on the mileage of the vehicle divided into rolling and driving sets with recorded linear and quadratic regression models.

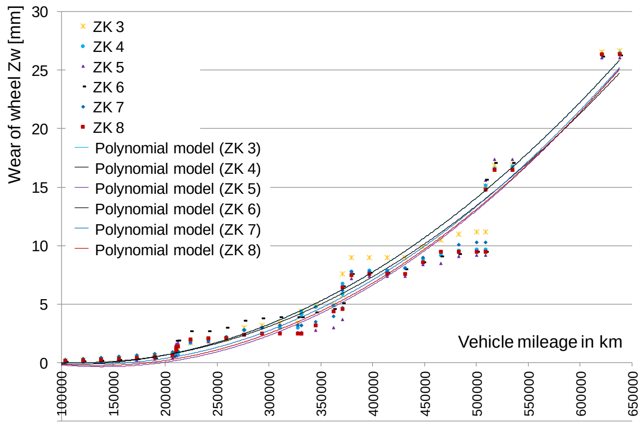

Fig. 12. Dependence of wear on the wheel diameter in the rolling circle D for the rolling sets ZK 3, ZK 4, ZK 5, ZK 6, ZK 7 and ZK 8 as a function of the vehicle mileage with regression quadratic models

The wear models on the diameter of the drive gears as a function of mileage are presented by the following equations:

$$Zw_{ZK1\_N} = 4.87 \cdot 10^{-11}x^2 + 2.07 \cdot 10^{-5}x - 2.06 \cdot 10^{-1} \quad (14)$$

$$Zw_{ZK2\_N} = 4.86 \cdot 10^{-11}x^2 + 2.08 \cdot 10^{-5}x + 3.81 \cdot 10^{-1} \quad (15)$$

$$Zw_{ZK9\_N} = 2.74 \cdot 10^{-11}x^2 + 3.44 \cdot 10^{-5}x - 1.02 \quad (16)$$

$$Zw_{ZK10\_N} = 2.44 \cdot 10^{-11}x^2 + 3.62 \cdot 10^{-5}x - 1.06 \quad (17)$$

the wear patterns on the diameter of the rolling set wheels are:

$$Zw_{ZK3\_T} = 9.28 \cdot 10^{-11}x^2 - 2.01 \cdot 10^{-5}x + 9.42 \cdot 10^{-1} \quad (18)$$

$$Zw_{ZK4\_T} = 9.28 \cdot 10^{-11}x^2 - 2.01 \cdot 10^{-5}x + 9.43 \cdot 10^{-1} \quad (19)$$

$$Zw_{ZK5\_T} = 10.22 \cdot 10^{-11}x^2 - 2.83 \cdot 10^{-5}x + 1.58 \quad (20)$$

$$Zw_{ZK6\_T} = 9.07 \cdot 10^{-11}x^2 - 2.09 \cdot 10^{-5}x + 1.19 \quad (21)$$

$$Zw_{ZK7\_T} = 9.78 \cdot 10^{-11}x^2 - 2.51 \cdot 10^{-5}x + 1.46 \quad (22)$$

$$Zw_{ZK8\_T} = 9.99 \cdot 10^{-11}x^2 - 2.68 \cdot 10^{-5}x + 1.53 \quad (23)$$

The coefficient of determination for the models described by equations (14)–(23) of wheel diameter wear as a function of mileage was 0.94–0.98. Based on the determined regression models of wheel wear, an attempt was made to estimate the mileage to reach the limit wear for wheels of 70 mm. For this purpose, systems of equations were derived for all regression models (14)–(23) and functions of constant coefficients being the dimension of limit wear. Table 7 presents the results of estimated mileages for each wheelset for the maximum limit wear of wheels after applying regression models of wheel diameter change and models of wheel wear on diameter D.

$$\delta = \frac{|x - x_z|}{x} \cdot 100\% \quad (24)$$

where: x – mileage value from the model of parameter change D, x_z – mileage value from the model of consumption D.

Table 7. Estimated values of the mileages that the electric multiple unit would achieve when using sets up to the limit dimension based on regression models of the change in diameter D and wear Zw with a relative percentage error

Wheelset	Vehicle mileage based on the diameter change model D	Vehicle mileage based on the wear model Zw	Relative percentage error
ZK 1	1010534.04	1002822.78	0.8
ZK 2	1011626.40	1001008.64	1.0
ZK 3	980943.23	977836.17	0.3
ZK 4	984734.16	977836.17	0.7
ZK 5	**971012.40**	**968052.05**	0.3
ZK 6	996818.29	993700.32	0.3
ZK 7	978439.23	975426.42	0.3
ZK 8	975848.10	972857.78	0.3
ZK 9	1105246.53	1078296.60	2.4
ZK 10	1124600.34	1095632.43	2.6

Additionally, in accordance with the relationship (24), the relative percentage error was determined for both models based on the change in diameter D and wear as a function of mileage [15].

Analyzing the results of the estimated mileage values for all wheelsets based on two types of regression models (based on the change in diameter and based on wear), which are presented in Table 7, it was found that in both cases of the models, convergent and satisfactory results were obtained. The average relative percentage error for both models was 1% in the case of wheel diameter. The models can be used interchangeably in the future determination of the mileage of wheelsets.

5. Conclusions

The analyses carried out on a group of the same vehicles showed that:

1) Wear on the rolling diameter of the drive wheelsets (extreme bogies) is greater than on the middle rolling sets. This is due to the fact that traction motors cause micro slippages during starting and electrodynamic braking. In relation to the wheelsets, which are the first to run onto the rail when driving in a curve and the fact of starting, the wear on the rolling circle is greater in relation to the rolling sets.
2) Observation of the maximum and minimum diameter dimensions during periodic P2 inspections allows for the creation of distributions of the relative frequency of occurrence of the maximum and minimum dimensions on individual wheelsets. This allows for better and faster visualization, and indirectly also informs about the speed of wheel wear.
3) Regression non-linear models can be used to model both the change in wheel parameters as a function of mileage (or time) and wear, and the criterion for model selection results from the highest value of the determination coefficient.
4) A quadratic regression model best describes changes in the diameter of wheelsets and wheel wear.
5) Determination of the approximate mileage that a wheelset would achieve if used to the limit dimension can be performed using either a model of diameter change as a function of mileage or a wear mode. In the case of wheel diameters, the differences between the obtained mileage values do not exceed 1%.
6) From a practical point of view, a model based on the results of changing a given parameter obtained from the P2 inspection is less laborious than the wear model, mileage results are obtained faster with fewer mathematical operations.

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Nomenclature

D	wheel diameter in rolling circle
DSU	Vehicle Maintenance System Documentation
ED	electrodynamic brake
ELF	electric low floor
ELM	Extreme Learning Machine
EN	electric low-floor traction unit
EP	electropneumatic brake
I_{OZ}	volumetric wear resistance
KE	brake from Knorr-Bremse
P2	periodic inspection second
P4	revision repair
PSO	Particle Swarm Optimization

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